

國立中央大學數學系

專題演講

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演講題目：**Nonlinear Elimination Preconditioned Space-Time
Algorithm for Hyperbolic PDEs: Insights and
Explanation Using ODE Theory**

演講時間：2024年06月18日(星期二) 11:00 a.m. ~ 12:00 p.m.

演講地點：中央大學鴻經館M107

Abstract：

As the first focus of this work, we aim to develop an efficient and robust solution algorithm for solving hyperbolic partial differential equations exhibiting solution discontinuity and highly local nonlinearity behavior. Such problems arise in numerous applications in computational science and engineering. Traditionally, parallel solution algorithms using time-marching methods focused on parallelizing computations within each time step while maintaining a sequential approach between time steps. Such a sequential step is the potential bottleneck of parallel algorithm performance. On the other hand, with the increasing computational power of modern parallel computer systems, there has been a growing interest in fully coupled space-time solution algorithms for time-dependent partial differential equations (PDEs), enabling parallelism in the temporal domain. To make the solution algorithm practical, it is crucial to have a robust and efficient nonlinear solver for large, sparse systems of equations in space-time algorithms. The inexact Newton with backtracking (INB) method is a well-known approach for solving nonlinear equations, but it exhibits slow convergence in the presence of shocks of our interests. In this work, we propose a new variant of nonlinear elimination preconditioning techniques to enhance the robustness and improve the efficiency of INB tailored to the solution characteristics of hyperbolic PDEs. By employing nonlinear elimination as a right or left preconditioner in conjunction with INB, we introduce two methods: INB with adaptive nonlinear elimination (INB-ANE) and nonlinear elimination preconditioned inexact Newton (NEPIN). These methods aim to reduce the impact of highly local nonlinearity and provide a better initial guess for the next Newton iteration. Numerical results show that the modification enhances the robustness of INB for Burgers' and Buckley-Leverett equations and reveal that NEPIN outperforms INB-ANE in identifying the correct shock location and introducing less interface pollution after the subspace correction before the global update. Additionally, the number of Newton iterations required to converge for NEPIN is almost independent of the time step and mesh size.

Although nonlinear preconditioning has been shown numerically to be a valuable technique to enhance the robustness and efficiency of Newton-type methods, a comprehensive theoretical or heuristic understanding of this technique still needs to be completed. Therefore, the second focus of this thesis is to introduce a new metric known as the stiffness ratio, which is borrowed from stiff ordinary differential equations (ODEs) to quantify the unbalanced nature of nonlinear systems. This metric is analogous to the condition number of the coefficient matrix for linear systems, which is used to assess matrix quality. Such insights offer valuable guidance and analytical tools for developing preconditioners tailored to specific applications. An investigation of a highly nonlinear algebraic system reveals that NEPIN reduces stiffness compared to INB. To illustrate this concept, we present numerical examples demonstrating the reduction of stiffness ratios of the corresponding Newton ODE system through nonlinear elimination.

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